Ser. No. EDU 001/17

GARISSA UNIVERSITY

UNIVERSITY EXAMINATION 2017/2018 ACADEMIC YEAR ONE FIRST SEMESTER EXAMINATION

SCHOOL OF EDUCATION, ARTS AND SOCIAL SCIENCES

FOR THE DEGREE OF BACHELOR OF EDUCATION (ARTS)

COURSE CODE: MAT 110 / MAT113

COURSE TITLE: BASIC / DIFFERENTIAL CALCULUS

EXAMINATION DURATION: 3 HOURS

DATE: 04/12/17 **TIME: 09.00-12.00 PM**

INSTRUCTION TO CANDIDATES

- **The examination has SIX (6) questions**
- **Question ONE (1) is COMPULSORY**
- **Choose any other THREE (3) questions from the remaining FIVE (5) questions**
- **Use sketch diagrams to illustrate your answer whenever necessary**
- **Do not carry mobile phones or any other written materials in examination room**
- **Do not write on this paper**

This paper consists of FOUR (4) printed pages ***please turn over***

QUESTION ONE (COMPULSORY)

(a) Evaluate the following limits:

i. $\lim_{x \to 4} \left\{ \dfrac{x^2 - 5x + 4}{x^2 - 16} \right\}$ **[3 Marks]**

ii. If $(x + y) = \sin(x + y)$, show that $\dfrac{dy}{dx} = -1$ **[4 Marks]**

(b) Find the derivatives of the following

i. $y = \sqrt{(1 + x^2)}$ **[3 Marks]**

ii. $y = x^{3x+1}$ **[3 Marks]**

iii. $y = \cos^{-1} 2x$ **[3 Marks]**

(c) If $y = \cos 2t$ and $x = \sin t$, find the equation of the normal to the curve at

$t = \dfrac{\pi}{6}$ **[4 Marks]**

(d) A ladder 13 meters long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at a rate of 3 meters per second. How fast is its height on the wall decreasing when the foot of the ladder is 5 meters away from the wall **[5 marks]**

QUESTION TWO

(a) Find the interval in which the function $f(x) = x^3 - 6x^2 + 3x - 5$ is concave up and concave down. **[4 Marks]**

(b) Find a point on the graph $y = 2x^3$ where the tangent is parallel to the chord joining $(1, 2)$ and $(3, 54)$ using mean value theorem. **[5 Marks]**

(c) Given that $f(x) = \dfrac{px + q}{x + 2}$, $\lim_{x \to 0} \{f(x)\} = 3$ and $\lim_{x \to \infty} \{f(x)\} = 4$, find the value of $f(-3)$

[6 Marks]

QUESTION THREE

(a) Differentiate $y = 3x^2 - 2$ from the definition of a derivative or using the first principles. **[3 Marks]**

(b) Find the gradient of the curve, $x = \frac{t}{1+t}$, $y = \frac{t^3}{1+t}$ at the point $\left(\frac{1}{2}, \frac{1}{2}\right)$ **[6 Marks]**

(c) Find the equation of the tangent and normal to the curve $x^3 + xy^2 + y^3 - 11 = 0$ at the point $x = 2, y = 1$ **[6 Marks]**

QUESTION FOUR

(a) The perimeter of a triangle is 10cm. If one of the sides is 4cm, what are other the other two sides for maximum area of the triangle **[4 Marks]**

(b) Find the maximum and minimum values of the function $y = \frac{(x-1)(x-6)}{(x-10)}$ and distinguish them. **[7 Marks]**

(c) Find the point of inflexion on the graph of the function $y = x^4 - 54x^2 - 2x$. **[4 Marks]**

QUESTION FIVE

(a) Differentiate

i. $y = \frac{\sin x}{x^2 \cos 2x}$ **[4 Marks]**

ii. (The radius of a variable sphere is increasing at the rate of 3cm per second. How fast is volume of the cube increasing when the radius is 10cm long **[3 Marks]**

(b) A window is in the form of a rectangle, surmounted by a semi-circle. If the perimeter of the window is to be 30 meters, find the dimensions so that the greatest possible amount of light may be admitted **[8 Marks]**

QUESTION SIX

(a) Differentiate $y = a^x$ **[3 Marks]**

(b) Verify mean value theorem for the function $f(x) = (x-1)(x-2)(x-3)$ in the interval $[0,4]$ and find c. **[5 Marks]**

(c) It is given that for the function $f(x) = x^3 - px + qx + 5$ on [1,3], Rolle's theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$. Find the values of p and q. **[7 Marks]**