Ser. No. IS 122/18

GARISSA UNIVERSITY

UNIVERSITY EXAMINATION 2017/2018 ACADEMIC YEAR ONE SECOND SEMESTER EXAMINATION

SCHOOL OF BUSINESS AND ECONOMICS

FOR THE DEGREE OF BACHELOR OF BUSINESS MANAGEMENT

COURSE CODE: ECO 113

COURSE TITLE: INTRODUCTION TO MATHEMATICS II

EXAMINATION DURATION: 3 HOURS

DATE: 11/04/18 **TIME: 9.00-12.00 PM**

INSTRUCTION TO CANDIDATES

- **The examination has SIX (6) questions**
- **Question ONE (1) is COMPULSORY**
- **Choose any other THREE (3) questions from the remaining FIVE (5) questions**
- **Use sketch diagrams to illustrate your answer whenever necessary**
- **Do not carry mobile phones or any other written materials in examination room**
- **Do not write on this paper**

This paper consists of FOUR (4) printed pages ***please turn over***

QUESTION ONE (COMPULSORY)

a) If $f(x) = 5^{2x+2x}$, find all the values of x such that $f(x) = 125$ **[3 marks]**

b) The table below shows the values of function $f(x)$ at different points of x.

x	-1	0	1	2	3
$f(x)$	3/2	3	6	12	24

i. Giving a reason, state whether function is an exponential or not.

ii. If $f(x)$ is an exponential function, find an exponential function that models the data. **[4 marks]**

c) A manufacturer estimates that when x units of a particular commodity are produced, they can all be sold when the market price is $\boldsymbol{p}$ dollars per unit, where $\boldsymbol{p}$ is given by the demand function $P = 200e^{0.01x}$

i. Write down expressions for the total revenue (TR) function

ii. How much revenue is obtained when 100 units of the commodity are produce **[4 marks]**

d) Find the range and the horizontal asymptote $f(x) = 1 - 2^{x+3}$ **[4 marks]**

e) Given that matrix $A = \begin{pmatrix} -0.5 & 0.75 & 0.75 \\ 1 & -1.5 & -0.5 \\ 0.5 & -0.25 & -0.25 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 3 \\ 0 & -1 & 2 \\ 2 & 1 & 0 \end{pmatrix}$

Show that $B = A^{-1}$ **[2 marks]**

f) Use the Gauss-Jordan method to find the inverse of matrix below:

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & -1 & 1 \\ 3 & 2 & 1 \end{pmatrix}$$

[6 marks]

g) The marginal revenue of a function is given by $MR = 25 - 8Q + 6Q^2 + 4Q^3$

Find the revenue function **[2 marks]**

QUESTION TWO

(a) A function $f(x)$ is defined by $f(x) = \frac{x}{x^2+1}$. Determine the intervals for which the function $f(x)$ is increasing or decreasing. **[6 marks]**

(b) Find the local maximum and the local minimum of the function

$$f(x) = 2x^3 - 21x^2 + 36x - 20$$ **[6 marks]**

(c) Find the area bounded by the curve $y = x^2$, the x-axis and the lines $x = 2$, and $x = 5$.

[3 marks]

QUESTION THREE

(a) Between 9:00 PM and 10:00 PM, cars arrive at Burger King's drive-thru at the rate of 12 cars per hour (0.2 car per minute). The following formula from statistics can be used to determine the probability that a car will arrive within $\boldsymbol{t}$ minutes of 9:00 PM.

$$F(t) = 1 - e^{-0.2\boldsymbol{t}}$$

Determine the probability that a car will arrive within 30 minutes of 9 PM.

What value does $\boldsymbol{F}$ approach as $\boldsymbol{t}$ increases without bound in the positive direction? **[3 marks]**

(b) Solve the equation

$2\ log_9(\sqrt{x}) - log_9(6x - 1) = 0$ **[4 marks]**

(c) Find all the real numbers x that satisfy the given equation.

$$\left(\frac{1}{9}\right)^{1-3x} = 3^{4x}$$

[4 marks]

(d) Find the domain and the vertical asymptote of asymptote $f(x) = ln\frac{1}{x-5}$ **[4 marks]**

QUESTION FOUR

Three persons A, B and C possess Sh.3000, Sh.2000 and Sh.2500 respectively. Person A with his entire amount purchased 5 shares of Sh. X each, 3 shares of Sh. Y each and 4 shares of Sh. Z each. Person B purchased 3 shares of Sh. X each, 4 shares of Sh. Y each and 2 shares of Sh. Z each with his entire amount and person C purchased 4 shares of Sh. X each, 3 shares of Sh. Y each and 4 shares of Sh. Z each with his entire amount. Determine the value of each share of different type. **[15 marks]**

QUESTION FIVE

(a) Find the determinant of the matrix below. **[3 marks]**

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{pmatrix}$$

(b) Solve the following system of equations by using Cramer's Rule. **[12 marks]**

$$x + y + z = 9$$

$$2x + 5y + 7z = 52$$

$$2x + y - z = 0$$

QUESTION SIX

(a) Given the demand function $P = 20 - 3Q$. Determine the revenue function and the marginal revenue function. **[4 marks]**

(b) A newly created state welfare agency wants to determine the number of analysts to hire to process the welfare application. Efficiency experts estimate the average cost C of processing an application is a function of the number of analyst (*x*). Specifically, the cost function is given by:

$$C = 0.001x^2 - 5\ln x + 60$$

Determine the number of analysts who should be hired in order to minimize the average cost per application. Show that the value obtained minimizes the cost. **[5 marks]**

(c) The demand function is given by $P = \frac{55}{\sqrt{Q}}$ and the cost of producing Q units is given by the function $C = 0.4Q + 700$. Find the price per unit (P) that gives the maximum profit **[6 marks]**