Ser. No. EDU 164/18

GARISSA UNIVERSITY

UNIVERSITY EXAMINATION 2017/2018 ACADEMIC YEAR ONE SECOND SEMESTER EXAMINATION

SCHOOL OF BIOLOGICAL & PHYSICAL SCIENCES

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: MAT 111

COURSE TITLE: GEOMETRY AND ELEMENTARY APPLIED MATHEMATICS

EXAMINATION DURATION: 3 HOURS

DATE: 10/04/18 **TIME: 09.00-12.00 PM**

INSTRUCTION TO CANDIDATES

- **The examination has SIX (6) questions**
- **Question ONE (1) is COMPULSORY**
- **Choose any other THREE (3) questions from the remaining FIVE (5) questions**
- **Use sketch diagrams to illustrate your answer whenever necessary**
- **Do not carry mobile phones or any other written materials in examination room**
- **Do not write on this paper**

This paper consists of TWO (2) printed pages ***please turn over***

QUESTION ONE (COMPULSORY)

(a) Find the focus and vertex of the parabola given by $x + y = -\frac{1}{2}x^2 + \frac{1}{2}$. **[5 marks]**

(b) Eliminate the parameter given that$tx = Sin\theta$and $y = Sin2\theta$ **[4 marks]**

Find a plane through $P_0(2,1,-1)$ and perpendicular to the line of intersection of the planes $2x + y - z = 3$and$dx + 2y + z = 2$. **[6 marks]**

(c) Prove that the distance d between a point $P_1(x_1, y_2, z_3)$ and the plane with the equation$Ax + By + Cz + D = 0$ is given by

$$d = \frac{|Ax_1 + By_2 + Cz_3|}{\sqrt{A^2 + B^2 + C^2}}$$ **[5 marks]**

(d) Prove that the diagonals of a parallelogram bisect each other. **[5 marks]**

QUESTION TWO

(a) Prove that the volume generated when the plane figure bounded by the polar curve$r = f(\theta)$ and the radius vectors at $\theta = \theta_1$ and $\theta = \theta_2$rotates about the initial line is given by $V = \int_{\theta_1}^{\theta_2} \frac{2}{3}\pi r^3 sin\theta d\theta$. **[9 marks]**

(b) Two boxes of mass$80kg$and$110kg$ are in contact and at rest on a horizontal surface. $A650N$push is exerted on the $80kg$box in the forward direction. If the coefficient of kinetic friction is 0.20, calculate the acceleration of the system and the force that each box exerts on one other. (Take$g = 9.81N/kg$) **[6 marks]**

QUESTION THREE

(a) By completing the square, find the centre and the radius whose equation is given by $x^2 + y^2 = 4x$. Obtain also its polar representation. **[5 marks]**

(b) Prove that the standard form of an equation of an ellipse, with centre (h, k) and major and minor axes of lengths $2a$ and$2b$ respectively, where $a > b$ is given by

$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ With a horizontal major axis **[10 marks]**

QUESTION FOUR

(a) Find the area enclosed by the curve $r=1+cos\theta$ and the radius vectors at $\theta=0$ and $\theta=\frac{\pi}{2}$.

[6 marks]

(b) Classify the following conics by use of the discriminant

i. $4xy-9=0$.

ii. $2x^2-3xy+2y^2-2x=0$.

iii. $x^2-6xy+9y^2-2y+1=0$. **[3 marks]**

(c) Identify the conic whose polar equation is given as $r=\frac{9}{5-4cos\theta}$. **[6 marks]**

QUESTION FIVE

(a) Find the area of the surface generated when the arc of the curve $r=ae^{\theta}$ between $\theta=0$ and $\theta=\frac{\pi}{2}$ rotates about the initial line. **[8 marks]**

(b) Find the parametric equations for the line in which the planes $3x-6y-2z=15$ and $2x+y-2z=5$ intersect. **[7 marks]**

QUESTION SIX

(a) Prove that the angle between two nonzero vectors$\boldsymbol{u}=\langle u_1,u_2,u_3\rangle$ and $\boldsymbol{v}=\langle v_1,v_2,v_3\rangle$ is given by

i. $\theta=cos^{-1}\frac{(u_1v_1+u_2v_2+u_3v_3)}{|\boldsymbol{u}||\boldsymbol{v}|}$. **(8marks)**

(b) A uniform ladder $10m$ long weighing $295N$rests against a smooth vertical wall with its base on a rough floor and $4m$from a wall. If the coefficient of friction between the ladder and the floor is 0.166, how far along the ladder will a $70kg$ man climb before the ladder slips from under him?

(Take $g=9.81N/kg$). **(7 marks)**