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**GARISSA UNIVERSITY**

**UNIVERSITY EXAMINATION 2019/2020 ACADEMIC YEAR TWO**

**SECOND SEMESTER EXAMINATION**

**SCHOOL OF SCHOOL OF PURE AND APPLIED SCIENCES**

**FOR THE DEGREE OF BACHELOR OF EDUCATION**

**COURSE CODE: STA 211**

**COURSE TITLE: PROBABILITY AND STATISTICS II**

**EXAMINATION DURATION: 2 HOURS**

**DATE: 11/12/2020 TIME: 03.00-05.00 PM**

**INSTRUCTION TO CANDIDATES**

* **The examination has FIVE (5) questions**
* **Question ONE (1) is COMPULSORY**
* **Choose any other TWO (2) questions from the remaining FOUR (4) questions**
* **Use sketch diagrams to illustrate your answer whenever necessary**
* **Do not carry mobile phones or any other written materials in examination room**
* **Do not write on this paper**

**This paper consists of SIX (6) printed pages *please turn over***

**QUESTION ONE (COMPULSORY)**

**QUESTION ONE (30 MARKS)**

1. The moment generating function of a random variable, $ X,$ is given by $M\_{X}\left(t\right)=1+2t+3t^{2}+4t^{3}+…,+.$ Determine the mean and variance of $X.$ (6 marks)
2. A continuous random variable, $X, $has probability density function given by $f\left(x\right)=\left\{\begin{array}{c}kx, 0\leq x\leq 10\\0, otherwise\end{array}\right.$ Find
3. The value of $k$ (2 marks)
4. $P\left(X\leq 6\right)$ (3 marks)
5. $E\left(X\right)$ (3 marks)
6. $Var(X)$ (3 marks)
7. The growth of a sunflower plant is found to be normally distributed with a mean of 10 and a variance of 7.5. Find the probability that a sunflower picked at random will have a height between 8m and 13m inclusive (4 marks)
8. A random variable $X$ has a Poisson distribution with mean of 4. Find $P(X\geq 10)$ (3 marks)
9. A random variable $X,$ has a binomial distribution with parameters $n=25$ and $p=0.27.$ Find $P(X\leq 1)$ (3 marks)
10. A discrete random variable $X,$ has the following probability distribution function

|  |  |  |  |
| --- | --- | --- | --- |
| $$x$$ | -1 | 0 | 1 |
| $$f(x)$$ | $$a$$ | $$b$$ | $$c$$ |

If $b=\frac{1}{2},$ determine $E(X^{2})$ (3 marks)

**QUESTION TWO (20 MARKS)**

1. A continuous random variable $X,$ has the probability distribution function, $f(x)$ given by

$$f\left(x\right)=\left\{\begin{array}{c}kx, 0\leq x<1\\k, 1\leq x<3\\k\left(4-x\right), 3\leq x<4\\0, otherwise\\\end{array}\right.$$

1. Find the constant $k$ (5 marks)
2. Find $P(0.5\leq X\leq 2.5$ (3 marks)
3. A fair coin is tossed three times. The number of sequences of heads is denoted by $X $and the number of sequences of tails is denoted by $Y.$ A sequence of heads begins and ends with a head so that for the outcome, $HHT, X=1 and Y=1$ and for the outcome $THT, X=1 and Y=2.$
4. List the sample space (1 mark)
5. Construct a table of the joint probability distribution of $X and Y$ (2 marks)
6. Find the marginal distribution of $X and Y$ (2 marks)
7. Show that $X and Y$ are not independent (4 marks)
8. Find $Cov(X,Y)$ (3 marks)

**QUESTION THREE (20 MARKS)**

1. Between 6pm and 7pm, safaricom directory enquiries receives calls at the rate of 2 per minute. Assuming that the calls arrive at random points in time, determine the probability that
2. 4 calls arrive in a randomly chosen minute (3 marks)
3. 6 calls arrive in a randomly chosen minute (4 marks)
4. A random variable, $X,$ has a Poisson distribution with mean 1.7. Determine $P(X>3)$ (3 marks)
5. (i) Given $X\~N(50,100)$, compute $P(45\leq X\leq 62)$ (5 marks)

(ii)Given $X\~N\left[76.5, \left(9.5\right)^{2}\right],$ if $P\left(X\geq k\right)=0.15,$ find $k.$ (5 marks)

**QQUESTION 4 (20 MARKS)**

1. The discrete random variable $X,$ has the distribution function

$$F\left(x\right)=\left\{\begin{array}{c}0, 0<-1\\\frac{1}{3}, -1\leq x<1\\\frac{1}{2}, 1\leq x<3\\\frac{5}{6}, 3\leq x<5\\1, x\geq 5\\, \end{array}\right.$$

Find

1. The probability distribution of $X, i.e f(x)$ (3 marks)
2. $P\left(X\leq 3\right)$ (2 marks)
3. $P\left(-0.4<X<4\right)$ (2 marks)
4. $Var(6X+11)$ (7 marks)
5. Mohamed catches a bus to work every morning. According to the timetable, the bus is due at 8am, but Mohamed knows that the bus can arrive at a random time between 5 minutes early and 9 minutes late. The random variable $X,$ represents the time in minutes after 7.55am when the bus arrives
6. Suggest a suitable model for the distribution of $X$ and specify it fully (1 mark)
7. Calculate the mean time of arrival of the bus (1 mark)
8. Find the cumulative distribution of $X$ (3 marks)
9. Mohamed will be late if the bus arrives after 8.05am. Find the probability that Mohamed is late for work (1 mark)

**QUESTION FIVE (20 MARKS)**

1. The discrete random variable $Y,$ has the probability distribution as follows

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| $$y$$ | 1 | 2 | 3 | 4 | 5 |
| $$P(Y=y)$$ | $$\frac{1}{15}$$ | $$\frac{2}{15}$$ | $$\frac{3}{15}$$ | $$\frac{4}{15}$$ | $$\frac{5}{15}$$ |

1. Write the probability generating function of $G\_{Y}(t)$ for the random variable $Y$ (3 marks)
2. Find the value of $G\_{Y}(t)$ at $t=1$ (2 marks)
3. Find $E(X)$ using $G\_{Y}(t)$ (2 marks)
4. An internet service provider has a large number of users connecting its computers. On average, only three users every hour fail to connect to the internet at their first attempt
5. Give two reasons why a Poisson distribution might be a suitable model for the number of failed connections every hour (2 marks)
6. Find the probability that in a randomly chosen hour,
7. All internet users connect at their first attempt (2 marks)
8. More than four users fail to connect at their first attempt (2 marks)
9. Write down the distribution of the number of users failing to connect at their first attempt in an 8-hour period (1 mark)
10. Using a suitable approximation, find the probability that 12 or more users fail to connect at their first attempt in a randomly chosen 8-hour period (5 marks)