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**GARISSA UNIVERSITY**

**UNIVERSITY EXAMINATION 2018/2019 ACADEMIC YEAR TWO**

**FIRST SEMESTER EXAMINATION**

**SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES**

**FOR THE DEGREE OF BACHELOR OF EDUCATION**

**COURSE CODE: MAT 212**

**COURSE TITLE: LINEAR ALGEBRA 1**

**EXAMINATION DURATION: 2 HOURS**

**DATE: 05/02/2020 TIME: 09.00-11.00 AM**

**INSTRUCTION TO CANDIDATES**

* **The examination has FIVE (5) questions**
* **Question ONE (1) is COMPULSORY**
* **Choose any other TWO (2) questions from the remaining FOUR (4) questions**
* **Use sketch diagrams to illustrate your answer whenever necessary**
* **Do not carry mobile phones or any other written materials in examination room**
* **Do not write on this paper**

**This paper consists of FOUR (4) printed pages *please turn over***

**QUESTION ONE (COMPULSORY)**

1. Define the following terms as used in Linear Algebra
2. Elementary matrix (1 mark)
3. Transpose of a matrix (1 mark)
4. Vector subspace, $T$ (2 marks)
5. Gauss-Jordan form of a matrix (2 marks)
6. Reduce the augmented matrix $\left[\begin{matrix}1&2&1\\3&1&2\\2&-4&-3\end{matrix} \begin{matrix}3\\-1\\7\end{matrix}\right]$ to

Echelon form and hence solve the equations

$$x+2y+z=3$$

$$3x+y+2z=-1 $$

$$2x-4y-3z=7$$

(6 marks)

1. $ a=2i+3j+4k$

$$b=i-2j+3k$$

Find the angle between $a$ and $b$ (5 marks)

1. Show that if $a, b $are vectors in $R^{3}$ then $a.\left(a×b\right)=0$ (5 marks)
2. Obtain the rank of the matrix $A$ and state whether it is invertible given that $A=\left[\begin{matrix}3&9&2\\1&5&6\\2&7&4\end{matrix}\right]$ (5 marks)
3. Prove that for any vector $x,y,z,\in R^{n}$ and any scalar $α \in $ $R^{n}$, $\left(x+y\right)z=xz+yz$(3 mark

**QUESTION TWO (20 MARKS)**

1. Apply the rank test to determine the nature of the solutions to the set of equations (9 marks)

$$x+2y-3z=1$$

$$x+3y+4z=2$$

$$2x+5y+z=3$$

1. Use $LU decomposition$ to solve the simultaneous equations (11 marks)

$$x+2y+3z=16$$

$$3x+5y+8z=43$$

$$4x+9y+10z=57$$

**QUESTION THREE (20 MARKS)**

1. Let $W\_{1}=\left\{\left(a,b,c\right):a=b=c\right\}, W\_{2}=\{\left(a,b,c\right):a=0\}$ be two subspaces of $R^{3}(R)$. Show that $R^{3}=W\_{1}⊕W\_{2}$ (3 marks)
2. Let $V=R^{3}$. Show that $W$ is a subspace of $V$ where

 $W=\left\{\left(a,b,c\right):a+b+c=0\right\}$ (4 marks)

1. Determine the three square elementary matrices corresponding to the operations : $R\_{1}\leftrightarrow R\_{2}, R\_{3}\rightarrow -7R\_{3}, R\_{2}\rightarrow R\_{2}-3R\_{3}.$ (3 marks)
2. Use Gaussian-Elimination method to solve the set of equations

$$2x-3y+2z=-9$$

$$3x+2y-x=4$$

$$x-4y+2z=6$$

**QUESTION FOUR (20 MARKS)**

1. (i) Define linear combination of vectors (2 marks)

(ii) Let $V\_{3}(R)$ be a vector space. Let $u=\left(1,7,-4\right), u\_{1}=(1,-3,2$ ) and $u\_{2}=(2,-1,1)$. Express $u$ as a linear combination of $u\_{1}$ and $u\_{2}$(4 marks)

1. (i) Define linear dependence (LD) of vectors (2 marks)

(ii) Show that the vectors $u\_{1}=\left(3,1,-4\right), u\_{2}=\left(2,2,-3\right), u\_{3}=(0,-4,1)$ in $V\_{3}(R)$ are linearly dependent (7 marks)

1. Show that $W=\{\left(x,y,5z\right):x,y,z \in R\}$ is a vector subspace of $R^{3}(R)$ (5 marks)

**QUESTION FIVE (20 MARKS)**

1. Suppose $U=\{\sin(x,\sin(2x,\sin(3x\})))$ is the set of real valued functions of $x.$
2. Show that $W\left(\frac{π}{4}\right)=1:W(x)$ is the Wroskian of $U.$ Hence or otherwise prove that $U is LD.$ (6 marks)
3. If $U, V $are linearly independent vectors in a vector space $Y,$ then prove that the set $\{U+V,2u-3v\}$ is LD. (5 marks)
4. A linear transformation $T$ is defined on the vector space of $R^{2}$ by $T\left[x,y\right]=[2x-y]$.
5. Give a rule for the inverse transformation $T^{-1}$ (4 marks)
6. Obtain $T[2b-c]$ (2 marks)
7. Show that $T $is linear (3 marks)