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**GARISSA UNIVERSITY**

**UNIVERSITY EXAMINATION 2018/2019 ACADEMIC YEAR THREE**

**SECOND SEMESTER EXAMINATION**

**SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES**

**FOR THE DEGREE OF BACHELOR OF EDUCATION**

**COURSE CODE: MAT310/MAT 216e**

**COURSE TITLE: REAL ANALYSIS I**

**EXAMINATION DURATION: 2 HOURS**

**DATE: 05/02/2020 TIME: 09.00-11.00 AM**

**INSTRUCTION TO CANDIDATES**

* **The examination has FIVE (5) questions**
* **Question ONE (1) is COMPULSORY**
* **Choose any other TWO (2) questions from the remaining FOUR (4) questions**
* **Use sketch diagrams to illustrate your answer whenever necessary**
* **Do not carry mobile phones or any other written materials in examination room**
* **Do not write on this paper**

**This paper consists of FOUR (4) printed pages *please turn over***

**QUESTION ONE (COMPULSORY)**

1. Let . Use the definition of an even integer to prove that is even if and only if is even. (5 Marks)
2. Show that the set of all polynomials with integral coefficients is countable (5 Marks)
3. (i) Give the definition of an open subset of and a closed subset of (2 Marks)

(ii) Let be given by . Show that is neither closed nor open. (3 Marks).

1. Give the definition of convergence of a sequence. Hence by considering the sequence show that (5 Marks)
2. Use the definition of uniform continuity to show that the function defined by is uniformly continuous (5 Marks).
3. Consider the equation .Use the Intermediate Value Theorem (IVT) to show that there is a real root to the equation between and. Find this root. (5 Marks).

**QUESTION TWO (20 MARKS)**

1. Define a metric on a set and use it to show that the standard metric for defined by for all is a metric. (8 Marks).
2. Let be a metric space and . Show that is closed in if and only if . (5 Marks).
3. (i)Give a brief description of the following terms as used in Mathematical Analysis: *interior point of a set* , *interior of a set* and the *boundary point of* , where (3 Marks).

(ii) For the set [. Find , , and where the symbols have got their usual meanings as used in Mathematical Analysis. (4 Marks).

**QUESTION THREE (20 MARKS)**

1. Prove that is irrational. (5 Marks)
2. Using the axioms and properties of an ordered field, prove that
3. [This is an identity called a difference of two squares] (5 Marks)
4. If and are negative, then is also negative. (2 Marks)
5. State the completeness property of and use it to prove that if is a closed and bounded subset of then and belong to (8Marks).

**QUESTION FOUR (20 MARKS)**

1. Give a brief description of a Cauchy sequence and use it to show that if is a sequence of real numbers such that is convergent, then it is Cauchy. (6 Marks).
2. Given , show that . Determine a value for associated with in accordance with the definition of limit of a function. (8Marks).
3. Show by Mathematical induction that the sequence defined by for all values of and is monotonic increasing and that for all . State giving reason(s) whether is convergent or divergent in . (6 Marks).

**QUESTION FIVE (20 MARKS)**

1. (i) Define a right limit and a left limit of a function using the definition. (2 Marks).

(ii) Let be defined by . Sketch the graph of and find and . State with reasons whether is a discontinuity of the first or second kind. (6 Marks).

1. Let be bounded on [a, b]. Prove that is Riemann integrable if and only if for each there exists such that . (8 Marks).
2. Show that the function be defined by is Riemann integrable. (4 Marks).