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**GARISSA UNIVERSITY**

**UNIVERSITY EXAMINATION 2018/2019 ACADEMIC YEAR FOUR**

**SECOND SEMESTER EXAMINATION**

**SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES**

**FOR THE DEGREE OF BACHELOR OF EDUCATION**

**COURSE CODE: MAT 410**

**COURSE TITLE: RINGS AND MODULES**

**EXAMINATION DURATION: 2 HOURS**

**DATE: 12/02/2020 TIME: 09.00-11.00 AM**

**INSTRUCTION TO CANDIDATES**

* **The examination has FIVE (5) questions**
* **Question ONE (1) is COMPULSORY**
* **Choose any other TWO (2) questions from the remaining FOUR (4) questions**
* **Use sketch diagrams to illustrate your answer whenever necessary**
* **Do not carry mobile phones or any other written materials in examination room**
* **Do not write on this paper**

**This paper consists of THREE (3) printed pages *please turn over***

**QUESTION ONE (COMPULSORY)**

1. If is a system satisfying all the conditions of a ring with unit element with possible exception of prove that the axiom must hold in and that is a ring. **(5 Marks).**
2. Define a nilpotent element of a ring and use a ring of of matrices with real entries to illustrate this. (2 Marks).
3. Prove that a commutative ring with unity is an integral domain exactly when for , , implies that , (4 Marks).
4. Give the definition of a left and right ideal and use it show that for a subring of the matrix ring , it is a right ideal but not a left ideal. (6 Marks).
5. Let and be the mapping that takes to , show that the mapping is a homomorphism and hence find its kernel. (7 Marks).
6. Prove that if is a ring with unit element, then is of characteristic if and only if and that is the smallest positive integer. (6 Marks).

**QUESTION TWO (20 MARKS)**

1. Show that for the operations, is an integral domain but not a field. (12 Marks).
2. If R is a ring such that for all prove that
3. all (3 Marks)
4. . (2 Marks)
5. is a commutative ring. (3 Marks)

**QUESTION THREE (20 MARKS)**

1. Define a ring homomorphism and use it to show that if where is a ring and the mapping defined by for all then is an isomorphism. (6 Marks).
2. Prove that if is a homomorphism with kernel S, then S is an ideal of (6 Marks).
3. Show that every finite integral domain is a field. (8 Marks).

**QUESTION FOUR (20 MARKS)**

1. Prove that for two ideals and of a ring , is an ideal of if and only if either or . (6 Marks).
2. Find the principal ideals of the ring (6 Marks).
3. In the ring show that is an ideal of (8 Marks).

**QUESTION FIVE (20 MARKS)**

1. By considering the ring where prove that is a unit if and only if (5 Marks).
2. Find the unit elements and zero divisors in the ring given in (a) above if (7Marks).
3. Use division algorithm to find the .Hence or otherwise, express this in the form . (8 Marks).