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**GARISSA UNIVERSITY**

**UNIVERSITY EXAMINATION 2018/2019 ACADEMIC YEAR FOUR**

**SECOND SEMESTER EXAMINATION**

**SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES**

**FOR THE DEGREE OF BACHELOR OF EDUCATION**

**COURSE CODE: MAT 412**

**COURSE TITLE: MEASURE THEORY**

**EXAMINATION DURATION: 2 HOURS**

**DATE: 07/02/2020 TIME: 2.00-4.00 PM**

**INSTRUCTION TO CANDIDATES**

* **The examination has FIVE (5) questions**
* **Question ONE (1) is COMPULSORY**
* **Choose any other TWO (2) questions from the remaining FOUR (4) questions**
* **Use sketch diagrams to illustrate your answer whenever necessary**
* **Do not carry mobile phones or any other written materials in examination room**
* **Do not write on this paper**

**This paper consists of THREE (3) printed pages *please turn over***

**QUESTION ONE (COMPULSORY)**

1. Let be the Lebesgue outer measure on and be a fixed set in such that Define for each by

Show that is also an outer measure on **(12 marks)**

1. Let and be subsets of with Show that **(6 marks)**
2. Define a Lebesgue measurable subset of

If is a non-Lebesgue measurable subset of show that there is a subset of such that **(12 marks)**

**QUESTION TWO (20 MARKS)**

1. Let with . Given that is any subset of such that show that (10 marks)
2. Let with . If there is a subset of with , show that **(10 marks)**

**QUESTION THREE (20 MARKS)**

1. (i) Define a measurable function, **(1 mark)**

(ii) Let Show that is measurable **(4 marks)**

1. Let where denotes the largest integer less than

and

Show that both and are measurable **(9 marks)**

1. Let be the characteristic function of a set . Prove that is measurable if and only if **(6 marks)**

**QUESTION FOUR (20 MARKS)**

1. Show that if is , then is also measurable. Moreover, if is real-valued, then is if and only if and are **(10 marks)**
2. Let a function be continuous. Show that is Lebesgue measurable. **(4 marks)**
3. If is measurable, then is also measurable. By use of an example, show that the converse is not necessarily true **(6 marks)**

**QUESTION FIVE (20 MARKS)**

1. State the Monotone Convergence theorem (MCT) **(4 marks)**
2. Given that use MCT to show that **(7 marks)**
3. Prove that  **(9 marks)**