****

**GARISSA UNIVERSITY**

**UNIVERSITY EXAMINATION 2018/2019 ACADEMIC YEAR FOUR**

**SECOND SEMESTER EXAMINATION**

**SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES**

**FOR THE DEGREE OF BACHELOR OF EDUCATION**

**COURSE CODE: MAT 413**

**COURSE TITLE: TOPOLOGY**

**EXAMINATION DURATION: 2 HOURS**

**DATE: 10/02/2020 TIME: 09.00-11.00 AM**

**INSTRUCTION TO CANDIDATES**

* **The examination has FIVE (5) questions**
* **Question ONE (1) is COMPULSORY**
* **Choose any other TWO (2) questions from the remaining FOUR (4) questions**
* **Use sketch diagrams to illustrate your answer whenever necessary**
* **Do not carry mobile phones or any other written materials in examination room**
* **Do not write on this paper**

**This paper consists of FOUR (4) printed pages *please turn over***

**QUESTION ONE (COMPULSORY)**

1. Give the definitions of the following terms as used in topology:
2. An **interior point** of a set .
3. An **accumulation point**.
4. A **closed set**.
5. The **closure** of a set.
6. A **dense subset** of a set. (5 Marks).
7. Given that defines a topology on a set and consider of . Determine whether and are limit points of . Hence find the derived set of . (5 Marks).
8. Consider the topology on a set . Find the closed subsets of and closures of and . Identify the dense subset(s) of (if any). (4 Marks).
9. Let be any set and for each , let be a subset of such that . Prove that . (3 Marks).
10. Briefly describe the union and intersection of a collection of a set Hence or otherwise, by letting and for each , let . Enumerate the sets , and and deduce the value of . (7Marks).
11. Prove that if is a metric space and a collection of all open subsets of , then
12. (1 Marks).
13. (1 Marks).
14. whenever  (4Marks).

**QUESTION TWO (20 MARKS)**

1. Describe briefly the meaning of the following terms as used in topology:
2. A topological space (3 Marks).
3. is strictly finer than (where and are topologies on ). (2 Marks)
4. Let be an infinite set and be a family of subsets of which include and all subsets of for which is finite. Show that is topology on . What happens to this topology if is a finite set? (9 Marks).
5. Let} be a family of topologies on . Show that is a topology on .Give a counter example to show that the union of a family of topologies is **not** a topology. (6 Marks).

**QUESTION THREE (20 MARKS)**

1. Define the term **base** of a topology  and hence show that is a base for a topology on exactly when for every and every  , there is a member such that (5 Marks).
2. Prove that for a non-empty set , a collection of all subsets of X is a base for a topology on X if and only if
3. .
4. then is a union of the members of (8 Marks).
5. Show that the family of all open spherical neighbourhoods of all points in a metric space is a base for a topology on . State the name of this topology. (7 Marks).

**QUESTION FOUR (20 MARKS)**

1. On , consider the class of all open intervals and b. Show that is a base for a topology on . (2 Marks).
2. Show that if is the usual topology on and is the upper limit topology on , then is strictly finer than . (5 Marks).
3. Prove that a set is closed if and only if it includes all its limit points, that is , where is the derived set of . (6 Marks).
4. Define a neighbourhood of a point and the neighbourhood system of Hence, by letting be a topological space, and be the neighbourhood system at , prove the following:
5. is not empty and every contains .
6. If then .
7. If and , then .
8. Every contains a such that G is a neighbourhood of its points. (7 Marks).

**QUESTION FIVE (20 MARKS)**

1. Give a brief description of the following terms as used in topology:
2. Continuous function. (1 Mark).
3. An **open** and a **closed** map. (2 Marks).
4. **Homeomorphic** topological spaces. (3Marks).
5. **First countable** topological space. (2 Marks).
6. **Hausdorff** space. (2 Marks).
7. Show that every metric space is a Hausdorff space. (10 Marks).