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**GARISSA UNIVERSITY**

**UNIVERSITY EXAMINATION 2018/2019 ACADEMIC YEAR FOUR**

**SECOND SEMESTER EXAMINATION**

**SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES**

**FOR THE DEGREE OF BACHELOR OF EDUCATION**

**COURSE CODE: MAT 413**

**COURSE TITLE: TOPOLOGY**

**EXAMINATION DURATION: 2 HOURS**

**DATE: 10/02/2020 TIME: 09.00-11.00 AM**

**INSTRUCTION TO CANDIDATES**

* **The examination has FIVE (5) questions**
* **Question ONE (1) is COMPULSORY**
* **Choose any other TWO (2) questions from the remaining FOUR (4) questions**
* **Use sketch diagrams to illustrate your answer whenever necessary**
* **Do not carry mobile phones or any other written materials in examination room**
* **Do not write on this paper**

**This paper consists of FOUR (4) printed pages *please turn over***

**QUESTION ONE (COMPULSORY)**

1. Give the definitions of the following terms as used in topology:
2. An **interior point** of a set $A$.
3. An **accumulation point**.
4. A **closed set**.
5. The **closure** of a set.
6. A **dense subset** of a set. (5 Marks).
7. Given that $τ=\{ X , ϕ , \left\{a\right\}, \left\{c,d\right\}, \left\{ a, c, d\right\} , \left\{b,c, d, e\right\}\}$ defines a topology on a set $X=\{a,b,c,d,e\}$ and consider $A=\{a,b,c\}$ of $X$. Determine whether $a,b,c,d$ and $e$ are limit points of $A$. Hence find the derived set of $A$. (5 Marks).
8. Consider the topology $ τ=\{ X , ϕ , \left\{a\right\}, \left\{a,b\right\}, \left\{ a, c, d\right\} , \left\{a, b,c, d\right\}, \left\{a, b,e\right\} \}$ on a set $X=\{a,b,c,d,e\}$. Find the closed subsets of $X$ and closures of $\left\{a\right\}, \left\{b\right\}$ and $ \left\{c,e\right\}$. Identify the dense subset(s) of $X$ (if any). (4 Marks).
9. Let $A$ be any set and for each $ p\in A$ , let $G\_{p}$ be a subset of $A$ such that $p\in G\_{p}⊂A$. Prove that $A=⋃\{G\_{p}: p\in A \}$. (3 Marks).
10. Briefly describe the union and intersection of a collection of a set $S . $Hence or otherwise, by letting $I=Z^{+}$ and for each $n\in Z^{+}$, let $A\_{n}=\{n\in Z :k\leq n\}$. Enumerate the sets $A\_{1}$, $A\_{2}$ and $A\_{3}$ and deduce the value of $ \bigcap\_{n=1}^{\infty }A\_{n}$ . (7Marks).
11. Prove that if $X$ is a metric space and $τ$a collection of all open subsets of $X$ , then
12. $ϕ\in τ$(1 Marks).
13. $X\in τ$(1 Marks).
14. $\bigcap\_{i=1}^{\infty }A\_{i}\in τ$whenever $A\_{i}\in τ$$∀$$ i=1,2,3….,n$ (4Marks).

**QUESTION TWO (20 MARKS)**

1. Describe briefly the meaning of the following terms as used in topology:
2. A topological space (3 Marks).
3. $τ^{\*}$ is strictly finer than $τ$ (where $τ^{\*}$ and $τ$ are topologies on $ X$ ). (2 Marks)
4. Let $X$ be an infinite set and $τ $be a family of subsets of $X$ which include $ϕ$ and all subsets $A$ of $X$ for which is finite. Show that $τ$ is topology on $X$. What happens to this topology if $X$ is a finite set? (9 Marks).
5. Let$ \{τ\_{α}: α\in Λ$} be a family of topologies on $X$. Show that $ \bigcap\_{α\in Λ}^{}τ\_{α}$ is a topology on $X$.Give a counter example to show that the union of a family of topologies is **not** a topology. (6 Marks).

**QUESTION THREE (20 MARKS)**

1. Define the term **base** of a topology $τ$ and hence show that $B$ is a base for a topology $τ$on $X$ exactly when for every $G\in τ$and every $x\in G$ , there is a member $ B\_{x}\in B$ such that $ x\in B\_{x}⊆G$ (5 Marks).
2. Prove that for a non-empty set $X$ , a collection $B$ of all subsets of X is a base for a topology on X if and only if
3. $⋃\left\{B: B⊆B\right\}=X$.
4. $B\_{1} , B\_{2} \in $ $B$ then $B\_{1}∩B\_{2}$ is a union of the members of $B$ (8 Marks).
5. Show that the family $B$ of all open spherical neighbourhoods of all points in a metric space $(X,q)$ is a base for a topology on $X$. State the name of this topology. (7 Marks).

**QUESTION FOUR (20 MARKS)**

1. On $R$ , consider the class $B$ of all open intervals $(a,b)$ $ a,b \in R $ and $a<$b. Show that $B$ is a base for a topology on $R$ . (2 Marks).
2. Show that if $u$ is the usual topology on $R$ and $τ\_{u}$ is the upper limit topology on $R$ , then $τ\_{u}$ is strictly finer than $τ$ . (5 Marks).
3. Prove that a set $E$ is closed if and only if it includes all its limit points, that is $E^{d}⊆E$ , where $E^{d}$ is the derived set of $ E$. (6 Marks).
4. Define a neighbourhood of a point $p $ and the neighbourhood system of $p.$ Hence, by letting $(X, τ)$ be a topological space, $p \in X $ and $ N\_{p}$ be the neighbourhood system at $p$, prove the following:
5. $N\_{p}$ is not empty and every $N \in $ $N\_{p}$ contains $p$.
6. If $M, N \in $ $N\_{p}$then $∩N \in $ $N\_{p}$.
7. If $M \in $ $N\_{p}$ and $N⊇M$ , then $ \in $ $N\_{p}$.
8. Every $N \in $ $N\_{p}$ contains a $G \in $ $N\_{p}$ such that G is a neighbourhood of its points. (7 Marks).

**QUESTION FIVE (20 MARKS)**

1. Give a brief description of the following terms as used in topology:
2. Continuous function. (1 Mark).
3. An **open** and a **closed** map. (2 Marks).
4. **Homeomorphic** topological spaces. (3Marks).
5. **First countable** topological space. (2 Marks).
6. **Hausdorff** space. (2 Marks).
7. Show that every metric space $(X,ρ)$ is a Hausdorff space. (10 Marks).