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**GARISSA UNIVERSITY**

**UNIVERSITY EXAMINATION 2018/2019 ACADEMIC YEAR THREE**

**SECOND SEMESTER EXAMINATION**

**SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES**

**FOR THE DEGREE OF BACHELOR OF EDUCATION**

**COURSE CODE: STA 301e**

**COURSE TITLE: PROBABILITY AND STATISTICS II**

**EXAMINATION DURATION: 2 HOURS**

**DATE: 10/02/2020 TIME: 09.00-11.00 AM**

**INSTRUCTION TO CANDIDATES**

* **The examination has FIVE (5) questions**
* **Question ONE (1) is COMPULSORY**
* **Choose any other TWO (2) questions from the remaining FOUR (4) questions**
* **Use sketch diagrams to illustrate your answer whenever necessary**
* **Do not carry mobile phones or any other written materials in examination room**
* **Do not write on this paper**

**This paper consists of FOUR (4) printed pages *please turn over***

**QUESTION ONE (COMPULSORY)**

1. A random variable $X$ can assume only odd integer values between 0 and 12. It is distributed in such a way that

 $f\left(x\right)=P\left(X=x\right)=\frac{x}{Σx}$

1. Find the probability distribution of $X$ (2 marks)
2. Find the $E(X)$ and the $Var(X)$ (5 marks)
3. Let $X $ be a random variable
4. Define the moment generating function of $X$ (1 mark)
5. If $X$ has a binomial distribution with $n$ trials and probability of success, $p,$ show that the moment generating function of $X$ is $M\_{X}(t)=(pe^{t}+q)$, where $q=1-p.$ (4 marks)
6. A discrete random variable $X$ has the following probability distribution:

|  |  |  |  |
| --- | --- | --- | --- |
| $$x$$ | -1 | 0 | 1 |
| $$f(x)$$ | $$a$$ | $$b$$ | $$c$$ |

1. If $b=\frac{1}{2},$ determine $E(X^{2})$ (3 marks)
2. If $b=\frac{1}{2}, E\left(X\right)=\frac{1}{6}$, determine the values of $a$ and $c$ (3 marks)
3. A random variable $X$ has its probability function given by

$f\left(x\right)=\left\{\begin{array}{c}c(\frac{2}{3})^{x}, x=1,2,3\\0, elsewhere\end{array}\right.$ Find the value of $c$ (3 marks)

1. Given that $X\~N\left(2, 25\right).$ Compute $P(-8<X<11)$ (3 marks)
2. The number of calls per 10 minutes received at safaricom switchboard follows a Poisson distribution with mean 0.6. Find the probability that
3. No calls will be received in the first 10 minutes (2 marks)
4. More than 2 calls will be received in a period of 40 minutes (4 marks)

**QUESTION TWO (20 MARKS)**

1. Find the probability function of a discrete non-negative random variable $X$ whose moment generating function is given by

$$M\_{X}(t)=(\frac{5}{8}+\frac{3}{8}e^{t})^{5}$$

Hence find the mean and variance of the random variable (3 marks)

1. Let $X$ denote a random variable having a Poisson distribution with mean, $λ=2$.

Find

1. $P(X=4)$ (4 marks)
2. $P(X\geq 4)$ (4 marks)
3. $P(X\geq 4/X\geq 2)$ (4 marks)
4. Let the moment generating function of a discrete random variable, $X$ be $M\left(t\right)=\frac{1}{6}e^{t}+\frac{2}{6}e^{2t}+\frac{3}{6}e^{3t}$. Find the $E(X)$ and the distribution of $X.$ (5 marks)

**QUESTION THREE (20 MARKS)**

1. A random variable $X$ has the following probability distribution function

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $$x$$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $$f(x)$$ | 0 | $$k$$ | $$2k$$ | $$2k$$ | $$3k$$ | $$k^{2}$$ | $$2k^{2}$$ | $$7k^{2}+k$$ |

1. Find $k$ and complete the table (6 marks)
2. Determine the cumulative distribution function of $X$ (2 marks)
3. A continuous random variable $X$ has probability density function given by

 $f\left(x\right)=\left\{\begin{array}{c}kx^{2}, 1<x<3\\0, otherwise\end{array}\right.$ .

 Determine

1. The value of $k$ (2 marks)
2. $P(X<2)$ (4 marks)

**QUESTION FOUR (20 MARKS)**

1. A random variable $X $ takes the values 1,2,3,4,5,6,7 which are mutually exclusive and mutually likely. Obtain the upper bound of

$$P\{\left|X-4\right|\geq 3\}$$

What is the exact probability? (11 marks)

1. If $X$ is the number scored in a throw of a fair die, show that the Chebychev’s inequality gives $P[\left|X-E(X)\right|>2.5]<0.47$ (9 marks)

**QUESTION FIVE (20 MARKS)**

1. The discrete random variable $Y,$ has the probability distribution as follows

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| $$y$$ | 1 | 2 | 3 | 4 | 5 |
| $$P(Y=y)$$ | $$\frac{1}{15}$$ | $$\frac{2}{15}$$ | $$\frac{3}{15}$$ | $$\frac{4}{15}$$ | $$\frac{5}{15}$$ |

1. Write the probability generating function of $G\_{Y}(t)$ for the random variable $Y$ (3 marks)
2. Find the value of $G\_{Y}(t)$ at $t=1$ (2 marks)
3. Find $E(X)$ using $G\_{Y}(t)$ (2 marks)
4. An internet service provider has a large number of users connecting its computers. On average, only three users fail to connect to the internet every hour at their first attempt.
5. Give two reasons why a Poisson distribution might be a suitable model for the number of failed connections every hour. (2 marks)
6. Find the probability that in a randomly chosen hour, all internet users connect at their first attempt (2 marks)
7. Find the probability that in a randomly chosen hour, more than four users fail to connect at their first attempt (2 marks)
8. Write down the distribution of the number of users failing to connect at their first attempt in an 8-hour period (1 mark)
9. Using a suitable approximation, find the probability that 12 or more users fail to connect at their first attempt in a randomly chosen 8-hour period (5 marks)